

# Distributed Nonlinear Model Predictive Control for Reheating Furnaces

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## INTRODUCTION

The strong demand for more advanced products and the growing focus on reduction of energy usage and emissions requires development of advanced process models and control systems. In Tata Steel R&D a new reheating furnace model and control system is developed based on Distributed Model Predictive Control (DMPC). The aim is more accurate calculation of the slab and furnace temperatures and optimal control in transient furnace conditions. Using model predictions and optimization gives the opportunity to anticipate on future events like roll changes, hot charging and planned delays.

The distributed optimizer is used to align multiple reheating furnaces with the scheduled mill pacing of the Hot Strip Mill. Dynamic furnace models are needed to predict the future of the process. The challenge in creating a dynamic reheating furnace model based on first principles, including full interaction between slabs and furnace, is the varying number, dimensions and positions of the slabs in the furnace. When the slabs move inside the furnace, the view factors and number of states change over time. Therefore, it is not possible to use a fixed grid to model the slabs in the furnace. This paper presents a calculation framework for state space models with varying state length. A set of ordinary differential equations (ODEs) is written in state space form and discretized using Tustin transformation. The linear state space models are allowed to have a non-square system matrix and come together with a mapping matrix that maps the states of previous timestep to the states of next timestep.

## REHEATING PROCESS

Reheating furnaces are used for heating steel slabs before hot rolling. The slabs are heated in a gas fired furnace to a temperature of around 1200°C. As long as the reheating furnaces are not the bottleneck in production, the interval times between discharging the slabs from the furnaces are decided by the rolling times in the mill. The control of the reheating furnace demands accurate modelling since the slab temperature cannot be measured reliably during the reheating process. The slabs are in an oxidizing environment where the oxide layer prevents accurate measurement of slab temperature. Besides accurate furnace models the control of the furnaces is challenging, since there are four furnaces running in parallel aiming to deliver the slabs at the aimed uniform temperature and with the right pacing for the mill. This has to be achieved with low emissions (CO<sub>2</sub>, NO<sub>x</sub>), minimal yield loss (surface oxidation) and for some products dissolution of precipitates. In Figure 1 the configuration of the reheating furnaces in Hot Strip Mill 2 in IJmuiden is shown.

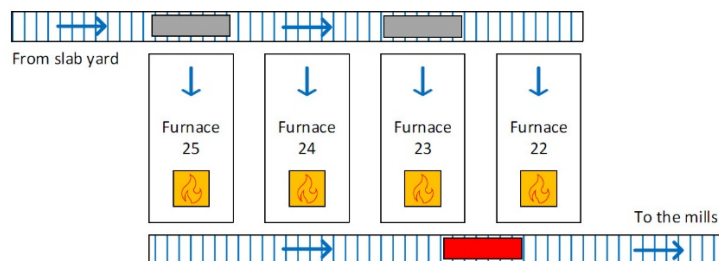


Figure 1: Reheating furnaces Tata Steel IJmuiden

### Mill and Furnace Interval Times

The slab schedule for rolling is primarily based on the quality and width of the slabs. For the furnace, pacing depends highly on slab width and target discharge temperature. When wide slabs are rolled, the speed of the furnaces constraints the total speed of the hot strip mill. This is because fewer wide slabs fit in the furnace, while the rolling time will be similar to narrow slabs. On the other hand, when narrow slabs are produced, the mill speed is the limiting factor of the hot strip mill. When the rolling scheme is known, the aim of the furnaces is to discharge the slabs exactly at the right time and temperature for rolling. When the hot strip mill is furnace constraint, the aim is to deliver the slab as quick as possible at the right temperature for rolling. The mill interval time is defined as the time between previous slab and the actual slab entering the mills and includes the known delays e.g. roll changes. The optimization objective for pacing is to follow mill interval time schedule as close as possible. A delta-interval-time is added for each slab to allow for furnace speed limitations. The optimizer will minimize the delta interval times to have to optimal speed of work. In table 1 an example of the rolling schedule is given. The sum of the mill interval times and delta interval times between two slabs discharged from the same furnace will result in the furnace interval time. For example, the colored blocks are adding up to the interval times for furnace 25.

Table 1: Example Rolling Schedule

Slab Index	Furnace Index	Mill Interval Time [s]	Delta Interval Time [s]	Furnace Interval Time [s]
0	Furnace 22	64	10	(depends on slabs before)
1	Furnace 23	68	8	(depends on slabs before)
2	Furnace 24	70	6	(depends on slabs before)
3	Furnace 25	68	5	299
4	Furnace 22	66	5	296
5	Furnace 23	65	3	288
6	Furnace 24	65	1	278
7	Furnace 25	70	0	275
8	Furnace 22	70	0	274
9	Furnace 23	360	0	566
10	Furnace 24	75	0	575
11	Furnace 25	80	0	585

### Reheating Furnace Layout

A schematic example of a furnace configuration is shown in Figure 2. Each reheating furnace consists of a convective zone and multiple burner zones where different types of gas can be combusted for heating the furnace and slabs. The hot flue gas of the burner zones will flow to the stack (see arrows), heating the cold slabs in the convective zone. The slabs are charged at the entry door (at the left side of the picture) and moving to the discharge door (at the right side of the picture). In the stack a recuperator exchanges heat between the flue gasses and the combustion air to recover the heat from the hot waste gasses. Each zone is equipped with thermocouples, measuring the wall and gas temperatures. These thermocouple measurements are used for guarding the maximum zone temperature and for adaptation of the thermal furnace model. The oxygen levels are measured at several positions in the furnace as well to monitor the combustion.

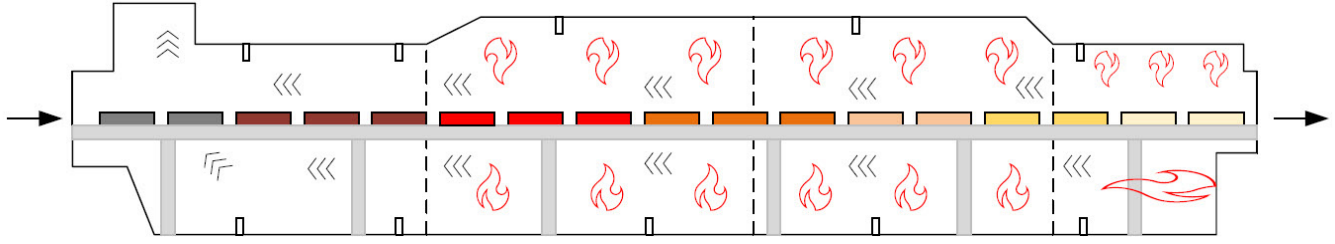


Figure 2: Schematic drawing of Reheating Furnace

### DYNAMIC FURNACE MODEL

Before (Distributed) Model Predictive Control can be applied, a dynamic furnace model is needed. The furnace models manipulated inputs are the gas flows at the burners and the slabs delta interval time. The model outputs are slab discharge temperature and thermocouple temperatures. Disturbances are the unplanned delays and variation in measured charging temperatures.

The radiation heat transfer is computed by the zonal model.<sup>[1]</sup> For analytical approximation of the view factors the furnace is divided in a set of cubic gas zones with volumes bounded by solid surfaces. For well-defined geometries the view factors can be calculated. The Weighted Sum of Grey Gases (WSGG) model is developed by Hottel and Sarofin in 1967<sup>[2]</sup>. The model describes the gas as a combination of grey gases where the absorption coefficient is considered constant. This greatly simplifies the integration in the spectral properties. The emittance in the WSGG models is introduced in Smith et al 1982<sup>[3]</sup>. The direct exchange area between any two zones/surfaces is generated by a simple summation of terms between component cubes and/or squares. Tables providing direct exchange areas between pairs of cubes, pairs of squares, and cubes and squares in close proximity to each other are available and approximation functions are used for the remaining pairs.

In earlier work the 'moving solid' approach was used to deal with the changing positions of the slabs<sup>[4]</sup>. In the new developed model all slabs are modelled individually. Advantage are more detailed modeling of the slabs and the possibility to add objectives for the slabs in the objective function of the controller. The furnace is divided into radiation zones where the interface between adjacent zones is solved by virtual planes, where the incoming heat rate equals the outgoing heat rate at the other side of the virtual plane ( $Q_{in} = Q_{out}$ ). The benefit of using virtual planes is a fixed matrix for the direct exchange areas and total exchange areas between the volumes and surfaces with simple geometries to calculate.

Besides the zonal model for radiation the first-principle furnace model includes:

- Combustion of several fuel types (e.g. Natural Gas (NG), Coke oven Gas (COG) and mixtures with blast furnace gas (BFG) or Hydrogen ( $H_2$ ))
- Gas flows in the furnace to the stack
- Skids: radiation (geometry) and conduction (thermal losses)
- Slab cells: material properties, radiation, convection and conduction
- Wall cells: material properties, radiation, convection and conduction

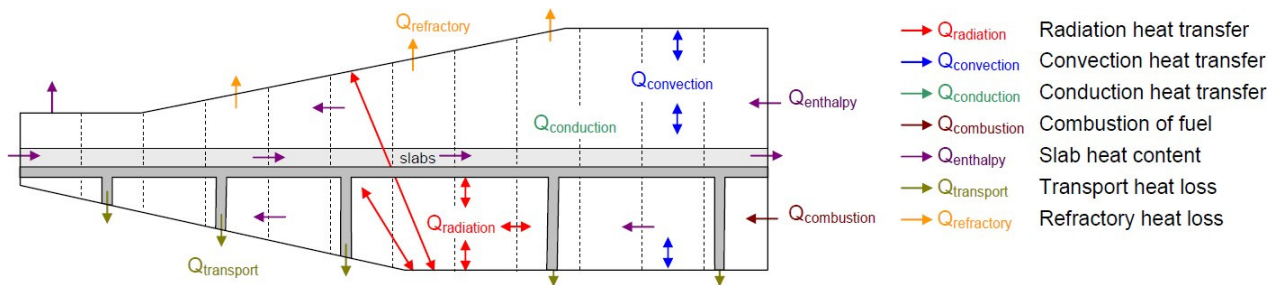


Figure 3: Schematic view of the heat transfer principles in a reheating furnace zone<sup>[4]</sup>

The heat rate vector  $Q(T, \phi)$  [W] describes the heat transfer for all surfaces and cells and is given as function of temperature  $T$  [K] and gas flow  $\phi$  [Nm<sup>3</sup>/h].

$$\begin{bmatrix} Q_a(T, \phi) \\ Q_b(T, \phi) \end{bmatrix} = \begin{bmatrix} Q_{cell_0}(T, \phi) \\ \vdots \\ Q_{cell_{N-1}}(T, \phi) \\ Q_{gas_0}(T, \phi) \\ \vdots \\ Q_{gas_{M-1}}(T, \phi) \\ Q_{surface_0}(T, \phi) \\ \vdots \\ Q_{surface_{S-1}}(T, \phi) \end{bmatrix} \quad (1)$$

For calculation of the temperature evolution over time the following differential equation is used. In this equation  $m$  is the mass of the cell [kg] and  $C_p$  is the specific heat of the material [J·kg<sup>-1</sup>·K<sup>-1</sup>]. The Hadamard product ( $\circ$ ) is used since the mass and specific heat are vectors.

$$\frac{\partial T}{\partial t} \circ m \circ C_p = Q_a(T, \phi) \quad (2)$$

Because surfaces have no mass, the net heat transfer at the surfaces will be zero:  $Q_{surface}(T, \phi) = 0$ . Since the gas dynamics are very fast compared to the slab and refractory dynamics, the gas is assumed to be in steady state, resulting in algebraic equations:  $Q_{gas}(T, \phi) = 0$ . The algebraic equations are eliminated from the set of differential equations. The system of equations is therefore split in a dynamic part ( $T_a, Q_a$ ) containing the partial derivatives and a steady part ( $Q_b$ ) containing the algebraic equations.

$$\begin{bmatrix} \frac{\partial T}{\partial t} \circ m \circ C_p \\ 0 \end{bmatrix} = \begin{bmatrix} Q_a(T, \phi) \\ Q_b(T, \phi) \end{bmatrix} \quad (3)$$

## LINEARISATION AND DISCRETISATION

For Model Predictive Control first the non-linear model is created based on first principles. The differential equations can be written in a nonlinear continuous state space model in the following standard form:

Standard non-linear state space model:

$$\frac{dx}{dt} = f(x, u) \quad y = g(x, u) \quad (4)$$

For the reheating furnace model, the states  $x$  are the temperatures in the furnace, the inputs  $u$  are the gas flows for the burner zones and the delta interval time and the outputs  $y$  are the discharge temperature and thermocouple measurements.

For the standard MPC framework, using a QP solver, the model needs to be linearized for each timestep in the operating point  $(x_s, u_s)$  of that moment. The non-linear state space model is linearized using the first order Taylor expansion:

$$f(x, u) \approx f(x_s, u_s) + \left. \frac{\partial f}{\partial x} \right|_{(x_s, u_s)} (x - x_s) + \left. \frac{\partial f}{\partial u} \right|_{(x_s, u_s)} (u - u_s) \quad (5)$$

$$g(x, u) \approx g(x_s, u_s) + \left. \frac{\partial g}{\partial x} \right|_{(x_s, u_s)} (x - x_s) + \left. \frac{\partial g}{\partial u} \right|_{(x_s, u_s)} (u - u_s) \quad (6)$$

This results for each timestep of the prediction horizon in a linear continuous time state equation and output equation:

$$\dot{x} = A_c x + B_c u + X_c \quad (7)$$

$$y = C_c x + D_c u + Y_c \quad (8)$$

The two terms  $X_c$  and  $Y_c$  are tracking the operating point wherein the model is linearized:

$$X_c = f_s - (A_c x_s + B_c u_s) \quad (9)$$

$$Y_c = g_s - (C_c x_s + D_c u_s) \quad (10)$$

For calculation of the timesteps the model is discretized using the Tustin Transformation. Because the prediction horizon is a series of models, where in each model the slabs are fixed at a certain position in the furnace, speed is not yet part of the equations. In discrete time it is possible to add the delta interval time as an input to the model. The time that the slab stays longer at a certain position is equivalent to the change of pacing of the furnace. The sampling time of the models in the prediction horizon can be initialized using the rolling interval times. When the process is furnace constraint the optimizer uses the delta interval time to slow down the pacing and achieving the slab target temperatures.

### DYNAMIC STATE LENGTH

In the furnace the amount of slabs can vary depending on the width of the slabs. In case a slab is charged or discharged, the size of  $x[k + 1]$  is different than  $x[k]$ . Therefore the standard MPC framework is extended to be able to deal with models with variable state length.

#### Mapping Matrix

The states can be mapped from the previous state vector to the new state vector using a mapping matrix  $M$ . This mapping matrix has the dimensions  $n \times m$ . The states from timestep  $k$  can be mapped to the new state index in timestep  $k+1$  using the mapping matrix  $M$ :

$$x[k + 1] = M \cdot x[k] \quad (11)$$

The dimensions of the  $A$  matrix are also  $n \times m$ .

$$x[k + 1] = A \cdot x[k] + B \cdot u[k] + X_s[k] \quad (12)$$

#### Discretization

Because  $x[k + 1]$  can have a different vector length than  $x[k]$ , the matrix  $A_c$  is a non-square matrix with dimensions  $m \times n$ . When discretizing, often Tustin transformation is used for preserving stability. Because it is not possible to invert the non-square matrix  $A_c$ , the formulation of Tustin transformation needs to be extended. For obtaining the discrete time matrices  $A$ ,  $B$  and  $X_s$ , the approximation will become:

$$\frac{dx}{dt} = A_c \left( \frac{1}{2} x(t) + \frac{1}{2} M^+ x(t + T_s) \right) + B_c \cdot u(t) + X_s \quad (13)$$

In this formulation  $M^+$  is the pseudoinverse of the mapping matrix  $M$ . When  $x[k+1]$  is longer than  $x[k]$  ( $m > n$ ),  $M$  has full column rank and its columns are independent. In that case the left inverse needs to be used:

$$M^+ = M_{left}^{-1} = (M^T M)^{-1} M^T \quad (14)$$

When  $x[k+1]$  is smaller than  $x[k]$  ( $m < n$ ), If  $M$  has full row rank, the rows of  $A$  are independent. the right inverse needs to be used:

$$M^+ = M_{right}^{-1} = M^T (M^T M)^{-1} \quad (15)$$

Defining the mapping matrix  $M$  as a matrix with full rank, only ones and zeros in the matrix and maximum one filled entry in each row and each column, the expression  $(M^T M)^{-1}$  is always the identity matrix. In that case the pseudoinverse of  $M$  is equal to  $M^T$ .

Tustin based approximation:

$$x[k+1] = \frac{M + \frac{T_s}{2} A_c}{I - \frac{T_s}{2} A_c M^T} x[k] + \frac{T_s}{I - \frac{T_s}{2} A_c M^T} (B_c \cdot u[k] + X_s) \quad (16)$$

### MODEL PREDICTIVE CONTROL

Model Predictive Control (MPC) is a control strategy that uses models to make predictions of the future process behavior. It optimizes the process variables over a particular timeframe in the future. MPC increases efficiency by pushing the process towards its process constraints without violating them and can anticipate on planned events.

Due to unmeasured disturbances, model errors and unexpected events, the ‘real’ future will probably deviate from the predicted future. Therefore, only for the first time step the optimized input is sent to the process and the measured output is used as new starting point of the next computation. The optimization is therefore repeated every sample time, which is in case of the reheating furnaces every minute.

To make sure all slabs in the furnaces are taken into account in the optimization problem, a finite horizon with at least one furnace load is used (35 timesteps). The prediction horizon is built from a series models. Each model describes the heat transfer and temperature evolution for the furnace with the slabs in a specific position. The next model in the prediction horizon is created with the new slab positions, assuming that one slab is discharged and eventually one or more slabs are charged based on the available space at the entry. The sampling time of each model is based on the interval time of the furnace.

Because the process is highly non-linear (e.g. radiation is to the fourth power), the non-linear model will be linearized in the operating point every time step and for every prediction step in the prediction horizon. The model is linearized based on the calculated state  $x[k+1]$  of the previous model in the prediction horizon and the optimized inputs from previous timestep. The discrete time linear models, together with the targets, constraints and actual states are input for the model predictive controller. The controller will predict and optimize the future and implement only the inputs of the first time step in the setpoints of the reheating furnace. An extended Kalman filter is used to estimate the thermocouple temperatures based on the measurements and previous (model) state estimation.

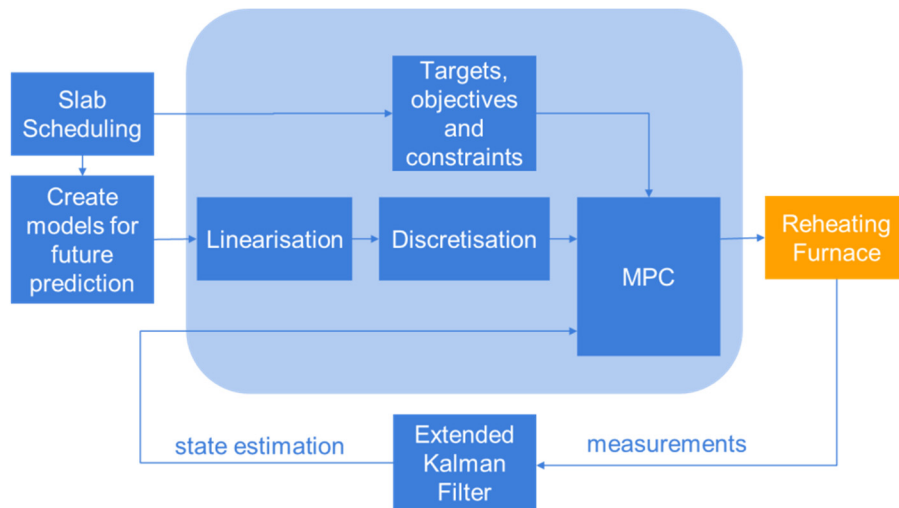


Figure 4: Schematic overview of Nonlinear Model Predictive Controller for Reheating Furnaces

## DISTRIBUTED MODEL PREDICTIVE CONTROL

Distributed Model Predictive Control (DMPC) enables the furnace MPC controllers to collaboratively and iteratively solve the synchronization challenge and determine the value of the shared interval time input. The Alternating Direction Method of Multipliers (ADMM) is used to solve the global optimization problem<sup>[5]</sup>.

Although the furnace speed is the shared input, the advantage of using DMPC compared to decentralized MPC will not result in an increased speed of work. Only a small advantage is seen in the speed of work since the slowest furnace primarily decides the pace of the reheating process. The main advantage of DMPC is better tracking of the slab temperature (less overheating) in the faster furnaces. When one furnace cannot keep up with other furnaces, it will dictate the overall pace. Because decentralized MPC has no knowledge of this, faster furnaces will optimize to unrealistically higher speeds. As a result, the slabs in the faster furnace may become overheated due to prolonged residence times. The application of Distributed MPC will therefore reduce the energy usage, yield loss by oxidation and overheating of slabs.

Initialization of the distributed MPC control is key to minimize the number of iteration needed before the problem has converged. After the first result of the individual furnace MPC optimization, the initialization of the distributed MPC can be done based on the maximum needed interval time for each timestep (slowest furnace).

## CONCLUSIONS

The reheating furnace model and Distributed Model Predictive Controller (DMPC) that was developed by Tata Steel R&D leads to more accurate calculation of the slab- and furnace temperatures and optimal control in transient furnace conditions. The future predictions gives the opportunity to anticipate on upcoming events like roll changes, hot charging and planned delays. The DMPC optimizes multiple reheating furnaces to match the scheduled mill pacing with minimal energy consumption and target slab temperatures.

A solution is presented for creating dynamic reheating furnace models based on first principles, including full interaction between slabs and furnace. When the slabs move inside the furnace, the view factors and number of states change over time. A new calculation framework is developed for the models with varying state length. A set of ordinary differential equations (ODEs) is written in state space form and discretized using Tustin transformation. The linear state space models can have a non-square system-matrix and come together with a mapping matrix that maps the states of previous timestep to the states of next timestep. This enables the use of the widely used state space and MPC framework for systems with variable state length.

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